A Preference Semantics for Imperatives

Will Starr

Cornell University
Sage School of Philosophy
will.starr@cornell.edu
http://williamstarr.net

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Outline

❶ Three Observations

❷ A Preference Semantics

Observation 1
Ross, Disjunction, Consequence and Imperatives

- If what (1a) says is true, then what (1b) says is true

  (1)   a. Kathy posted the letter
        b. Kathy posted the letter or Kathy burnt the letter

    - Classical semantics predicts this: $P \vDash P \vee B$

- Suppose that what (2a) commands is required
- Does it follow that what (2b) commands is required?

  (2)   a. Kathy, post the letter!
        b. Kathy, post the letter or burn the letter!

- Ross' (1944) Puzzle: $P \vDash P \vee B$ but $!P \nvDash !P \vee !B$
- Proposition being true $\neq$ command being required?

Observation 1
Ross and Imperative Consequence

- Proposition being true $\neq$ command being required?
- Maybe:
    - Propositions are true, commands are satisfied
- Then imperative consequence is satisfaction-preservation
- So maybe $!P \vDash !P \vee !B$
- Maybe talk of requirement was pragmatic noise...

Against Satisfaction Consequence

❶ Correct propositions are true
❷ Correct commands are?
  - Satisfied ×; Required ✓

Observation 1
Imperative Consequence is not about Satisfaction

Fact 1: !P ⊭ May B
- President's command:
  - (3) Will, post the letter!
- I cannot infer that
  - (4) I may burn the letter

Against Satisfaction Consequence
- If imperative consequence is about satisfaction:
  - !P ⊨ !P ∨ !B
- Consequence is transitive:
  - !P ⊨ May B × ×

Fact 2: !P ∨ !B ⊨ May B
- The president's command:
  - (5) Will, post the letter or burn the letter!
- I can infer:
  - (6) I may burn the letter

Observation 1
Foreshadowing

Generalized Consequence
- An agent which accepts the premises has implicitly accepted the conclusion
  - Declaratives After accepting premises, accepting conclusion provides no new information
  - Imperatives After accepting premises, accepting conclusion provides no new permission
- Different kinds of sentence, different kinds of acceptance

Consequence in Dynamic Semantics
The generalized definition can be formulated with a dynamic semantics (Veltman 1996)

Observation 2
Felicity, Context & Information

(7) # Unicorns don't exist. Bring me a unicorn!

(8) # The door is open. Open the door!

Relatedly:

(9) a. I don't have a brother.
     b. # If I had a brother, call him!

Generalization
The felicity of imperatives depends on the mutual information against which they are issued. Specifically, the possibility of the action they proffer must be open.

Bonus for: saying *why* imperatives are about open actions.

Observation 3
Imperatives Scope Under Connectives

(10) Go home and I'll go to the grocery store.
  a. **Assertion Conditional**:
     Go home! And **if** you do, I'll go to the store
  b. **Sequenced**:
     I'll go to the grocery store and you go home
  c. **Command Conditional**:
     If you go home, I'll go to the grocery store
     (And, you know what happens when I shop!)

- Sequenced requires imperative to scope under *and*
- Arguably, same point holds for conditional imperative:

(11) If you're sleepy, drink coffee!
(12) If Chris gets up, I'll call on him and you close the door.

Preference, Rationality & Context
Information

- Informational contents (*propositions*) are sets of possible worlds
  - These sets distinguish ways world might be (worlds in the set) from ways it isn't (worlds excluded from set)
- One informational content is particularly useful for understanding how linguistic interactions unfold:

Contextual Possibilities (c)

As communication and inquiry unfold, a body of information accumulates. Think of this information as what the agents are mutually taking for granted in some way. I call the set of worlds embodying this information c, short for *contextual possibilities.* (Stalnaker 1978; Lewis 1979)

Preference, Rationality & Context
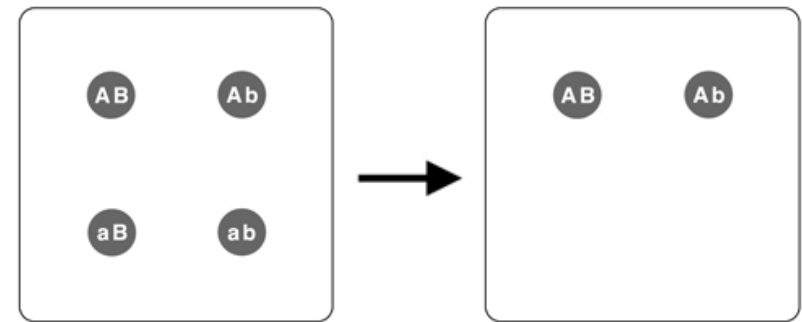## Information and the Process of Inquiry

Figure: Accepting the information that A

- Inquiry progresses by gaining information, i.e. the elimination of worlds.
- $\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\} \Rightarrow \{w_{AB}, w_{Ab}\}$

Preference, Rationality & Context
Issues

- It's not just information that accumulates in communication and inquiry (Bromberger 1966)
- There are issues (e.g. Hamblin 1958; Roberts 1996).
- They can be thought of as ways of grouping worlds in c into competing alternative propositions.

Alternatives (C) (e.g. Groenendijk 1999; Hulstijn 1997)

Alternatives represent open, competing propositions the agents are concerned with deciding between; their **issues**. Formally, this grouping of c may be identified with a set of sets of worlds; call it C. There is no need to also keep track of c: it is just the union of all the alternatives in C.

Preference, Rationality & Context
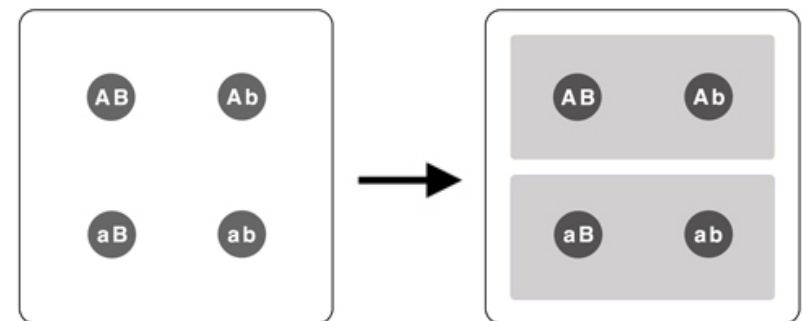## Issues and Inquiry

Figure: Recognizing the issue whether A

- Inquiry also progresses by recognizing issues, i.e. introducing alternatives
- $\{\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}\} \Rightarrow \{\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\}$

Preference, Rationality & Context
Preferences

- Agents not only gather information and identify competing alternatives, they form **preferences** regarding those alternatives
- Central to **decision theoretic** approaches to rational choice, as applied in philosophy, AI and economics (e.g. Ramsey 1931; Newell 1992)
- Of relevance here: the preferences being mutually taken for granted for the purposes of an interaction
  - Parallel to Stalnaker's common ground

Preference, Rationality & Context
Preferences

- A body of preferences can be represented as a binary **preference relation** on the alternatives
- I.e. a set of pairs of propositions constructed from c

Preference State (R)

- R: binary relation on alternatives (open propositions)
- $R(a, a')$: a is preferrable to a'
- Each pair in R is called a *preference*
- Set of (non-empty) alternatives over which R is defined: issues at stake in R, C_R
- Set of worlds among those alternatives: the contextual possibilities written c_R

Preference, Rationality & Context
Information in a Preference State
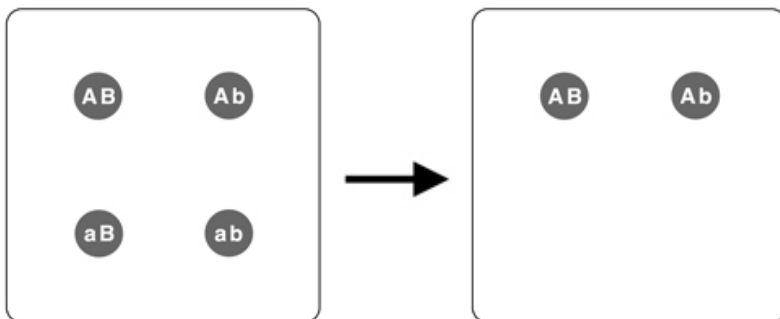

Figure: Accepting the information that A

- $\{w_{AB}, w_{Ab}, w_{aB}, w_b\} \Rightarrow \{w_{AB}, w_{Ab}\}$
- $\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_b\}, \varnothing\rangle\} \Rightarrow \{\langle\{w_{AB}, w_{Ab}\}, \varnothing\rangle\}$

Preference, Rationality & Context
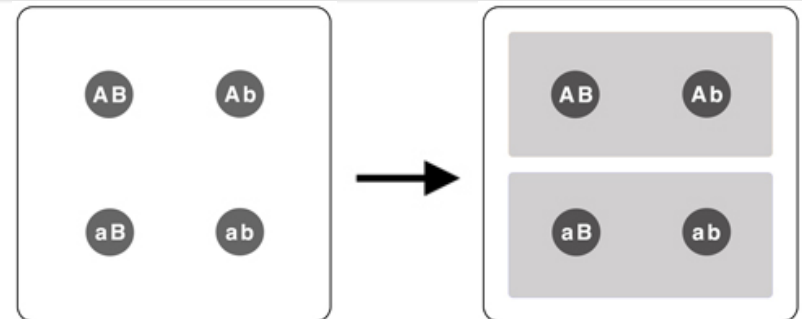Issues in a Preference State

Figure: Recognizing the issue whether A

- $\{\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}\} \Rightarrow \{\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\}$
- $\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle\}$
  $\Rightarrow \{\langle\{w_{AB}, w_{Ab}\}, \varnothing\rangle, \langle\{w_{aB}, w_{ab}\}, \varnothing\rangle\}$

Preference, Rationality & Context
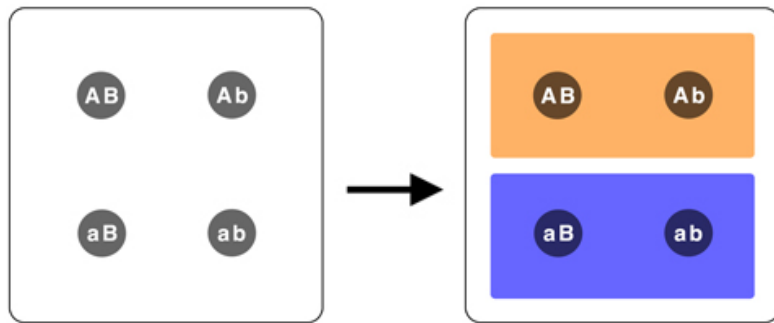## Preference and Inquiry

Figure: Coming to prefer A (to ¬A)

- $\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle\}$
  $\Rightarrow \{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle\}$

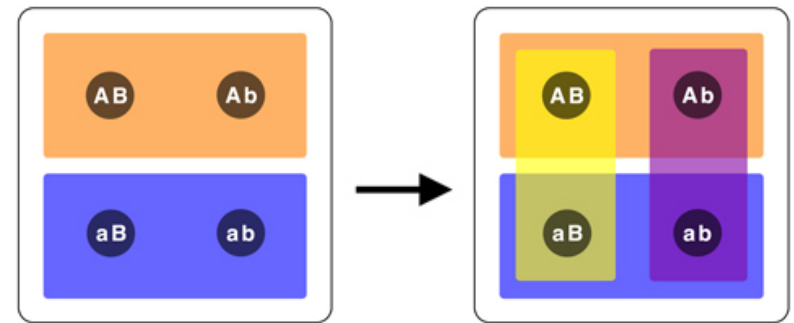# Preference, Rationality & Context
Preference and Inquiry

Figure: Adding (separate) preference for B to preference for A

- $\{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle\} \Rightarrow$
  $\{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle, \langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$

Preference, Rationality & Context
Using Preference to Make Rational Choices

- Given preference relation, which alternatives are best?
- How do you use preferences to decide what to do?
- In decision theory, this takes the form of defining a choice function (Hansson & Grüne-Yanoff 2009)
- A choice function Ch maps a preference state R to the set of best alternatives according to R

Proposal: Choice, Permission, Requirement
1. $Ch(R)$ are the alternatives permissible according to R
2. Required by R: unique alternative permitted by R

Preference, Rationality & Context
The Choice Function: Logical Weak Dominance

Which Alternatives are Best?
1. Competition between **preferred alternatives** $P(R)$
   - Left member in some pair
2. If preferred alternative a is entailed another preferred one, then a is out
3. If a entails a dispreferred alternative, a is out

Choice: Formally
$$Ch(R) = \{a \in P(R) \mid \nexists a' \in P(R) : a' \subset a$$
$$\& \ \nexists a' \in D(R) : a \subseteq a'\}$$

[$D(R)$: dispreferred alternatives]

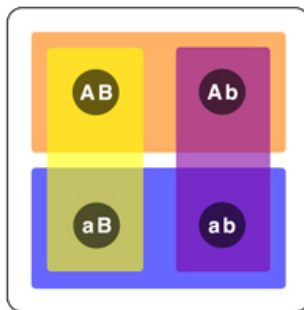# Preference, Rationality & Context
How Choice Works: An Example

Figure: Preference for A with (separate) preference for B

- $\{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle, \langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$
- Two **preferred** (warm) alternatives, orange and yellow
- Neither entails the other nor dispreferred (cold) alt.
- So $Ch(R) = \{\{w_{AB}, w_{Ab}\}, \{w_{AB}, w_{aB}\}\}$

Preference, Rationality & Context
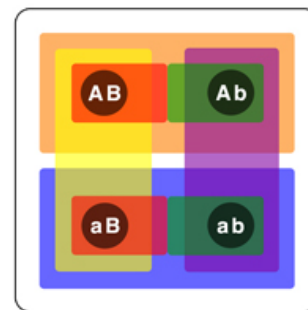## How Choice Works: A More Complex Example

- 4 pref. alt's: yellow, orange, reds
- Yellow is out: reds entail it
- Orange is out: top red entails it
- Bottom red is out: it entails blue, which is a dispreferred alt
- Unique best alternative: top red
- $A \wedge B$ is required

Figure: Pref A and B

$\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{aB}\}, \{w_{aB}, w_{ab}\}\rangle,$
$\langle\{w_{AB}\}, \{w_{Ab}\}\rangle, \langle\{w_{aB}\}, \{w_{ab}\}\rangle,$
$\langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$

Preference, Rationality & Context
What Must Preferences Be Like for Choice to Guarantee Results?

Exclusivity
- $\forall a, a' : a \cap a' = \varnothing$ if $R(a, a')$
- *When you strictly prefer one thing to another, the two can't be compatible.*

No Absurdity
- $\forall a \neq \varnothing : \langle a, \varnothing\rangle \in R$ & $\langle\varnothing, a\rangle \notin R$
- *Always prefer non-absurd alternatives to absurd one.*

Irreflexivity
- $\forall a : \langle a, a'\rangle \notin R$ if $a' \subseteq a$
- *You can't strictly prefer an alternative to something that entails it.*

Preference, Rationality & Context
These Constraints on Preferences are Pragmatic

Semantics, Pragmatics & Irrational Preferences
- Words can get us into irrational preference states
  - So none of these axioms are enforced in the semantics
- Rather, recognizing their satisfaction and frustration is part of pragmatics
- Grice: pragmatics is about general rational cooperation
- Decision Theory: rational agents follow preference axioms

The Semantics: some preliminaries
Radicals & Worlds

Radicals (Informational Core)

- Basic sentences: mood marker + radical, e.g. $!\rho$
  - Mood markers: $!, \rhd, ?$
  - Atomic radicals: A, B, C, etc.
  - Logically complex radicals: $\neg\rho, \rho_1 \wedge \rho_2, \rho_1 \vee \rho_2$

Worlds

A possible world is a function which maps atomic radicals to a unique truth-value, 1 or 0

- Dynamic Meaning: function from contents to contents
- $R[\phi] = R'$: R' is the result of applying ϕ to R (Veltman 1996)

The Semantics: atomic 'radical' semantics
Convenient fiction for handling sub-sentential disjunction/conjunction

Radical Semantics

- $c[A] = \{w \in c \mid w(A) = 1\}$, for any atomic radical A

- Subsentential semantics
- Filters alternatives for worlds where radical is true

Connective Semantics (Heim, Veltman)

- $c[\neg\rho] = c - c[\rho]$
- $c[\rho_1 \wedge \rho_2] = (c[\rho_1])[\rho_2]$
- $c[\rho_1 \vee \rho_2] = c[\rho_1] \cup c[\rho_2]$
- $R[\phi \wedge \psi] = (R[\phi])[\psi]$
- $R[\phi \vee \psi] = R[\phi] \cup R[\psi]$

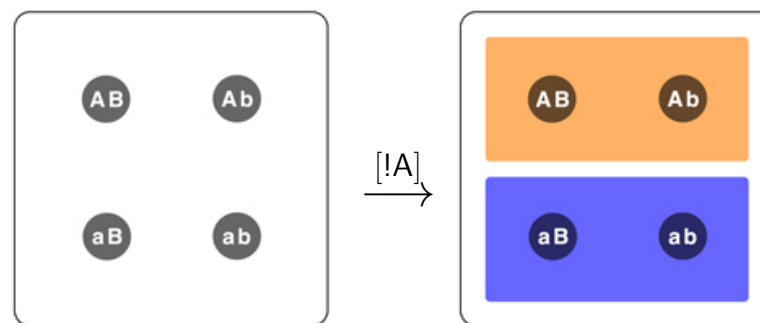(Negation never scopes over mood Han 2001 a.o.)

The Semantics
Imperative Semantics

Imperative Semantics

$R[!\rho] = R \cup \{\langle a[\rho], a - a[\rho]\rangle \mid a \in A_R\}$

- A_R: non-empty R-alternatives, plus their union c_R

This amounts to a three-step process:

1. Admit all of the preferences in R
2. **Local Preferences**: Take each incoming non-empty alternative a and introduce a preference for the ρ-worlds in a over the non-ρ-worlds in a
3. **Global Preference**: Introduce a preference for all of the ρ-worlds in c_R over the non-ρ-worlds

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The Semantics
A Simple Example

Figure: R to $R[!A]$

$R = \{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle\} \Rightarrow$
$\{\ \langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle\ \}$
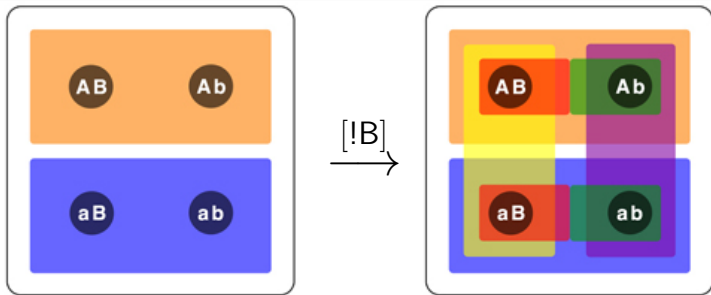
The Semantics
A Complex Example

Figure: $R[!A]$ to $R[!A][!B]$

$$\{ \langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle \} \Rightarrow$$
$$\{ \langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle,$$
$$\langle\{w_{aB}\}, \{w_{ab}\}\rangle, \langle\{w_{AB}\}, \{w_{Ab}\}\rangle,$$
$$\langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle \}$$
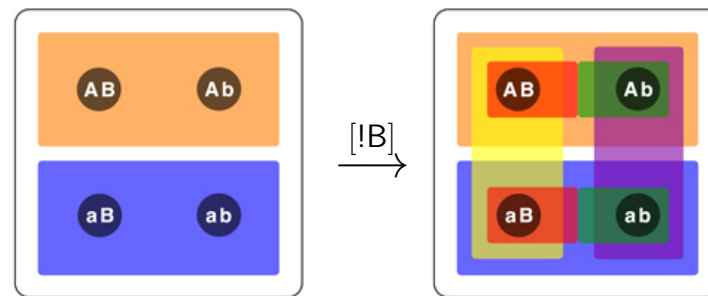

The Semantics
A Complex Example

Figure: $R[!A]$ to $(R[!A])[!B]$

- Recall $R[!A \wedge !B] = (R[!A])[!B]$
- So this is the interpretation of conjoined imperatives
- Let's mix in a declarative conjunct...

The Semantics: Observation 3
Mixing Moods

Figure: $(R[\triangleright G])[!H]$

- *I'll go to the grocery store and you go home*: $\triangleright G \wedge !H$
- $R[\triangleright G \wedge !H] = (R[\triangleright G])[!H]$

The Semantics: Observation 2
Felicity, Context and Information

Figure: $(R[\triangleright \neg U])[!B]$

- *Unicorns don't exist. Bring me a Unicorn*:
  - $(R[\triangleright \neg U])[!B]$
- Irrational preference: preferring the absurd!
  - Hence (pragmatically) infelicitous

The Semantics: Observation 1
We want $!A \nvDash !A \lor !B$

Informational Support & Consequence (Veltman)

- $c \vDash \phi \iff c[\phi] = c$
- $\phi_1, \ldots, \phi_n \vDash \psi \iff \forall c : c[\phi_1] \cdots [\phi_n] \vDash \psi$

Preferential Support & Consequence (Starr)

- $R \vDash \phi \iff Ch(R) = Ch(R[\phi])$
- $\phi_1, \ldots, \phi_n \vDash \psi \iff \forall R : R[\phi_1] \cdots [\phi_n] \vDash \psi$

- Both kinds of consequence and support are useful
- The first when tracking information
- The second when tracking the best alternatives

The Semantics: Observation 1
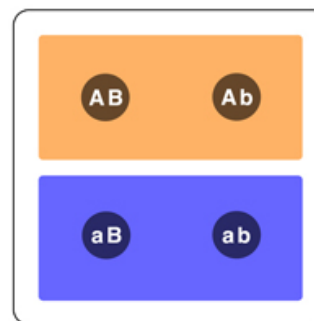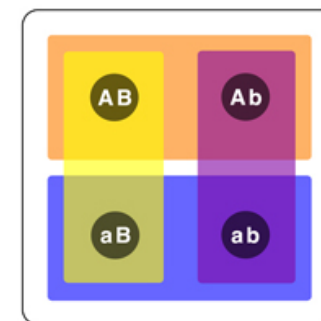### We want $!A \nvDash !A \lor !B$

Figure: $R(!A)$

Figure: $R(!A \lor !B)$

- Why: $!A \nvDash !A \lor !B$
  - Disjunctive imperatives create more permissions
- But consequence is about preserving permissions

Imperatives and Modals
Differences and Connections (Starr to appear)

- $!A$ makes A-worlds preferable to ¬A-worlds
  - Doesn't make all best alternatives contain only A-worlds if conflicting preference is present
  - No coherent interpretation for ¬$!A$
- Must A tests: *all* best alternatives *entail* A
  - ¬Must A: state doesn't pass this test
- May A tests: *some* best alternative is *compatible* w/A
  - ¬May A: state doesn't pass this test
- **Descriptive use**: 3rd person auth. has preferences w/this feature, make common preferences match
- **Performative use**: 1st person auth. has preferences w/this feature, make common preference match

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Illocutionary Variability
Semantic Effect $\neq$ Pragmatic Effect
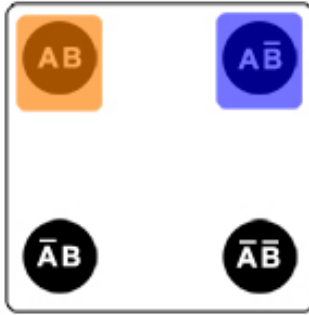
(13)
  a. Try the felafel! (*Advice*)
  b. Buy a new car today! (*Advertisement*)
  c. Have another beer! (*Permission*)
  d. Have a nice day!/Drop dead! (*Wish*)
  e. Leave your name at the tone (*Instruction*)
  f. Sit down! (*Command*)

- Imperatives introduce a **common preference**
  - The point and implications of doing so depend on **pragmatic** factors
- Pragmatic factors: how authoritative speaker's preferences are; presumption of opposite preference; whether signaling that preference is nice; whether having that preference will help hearer achieve goals

The Semantics
Conditionals and Conditional Imperatives

Conditional Semantics

$$R[(\text{if } \phi)\,\psi] = \{r \in (R \cup (R[\phi])[\psi]) \mid c_{R[\phi]} = c_{(R[\phi])[\psi]}\}$$

Figure: $R[(\text{if A})\,!\text{B}]$

Conclusion
Three Observations and a Semantics

Summary

1. Imperatives introduce preferences
2. Preferences are used to determine what's permitted/required
3. Imperative consequence: preservation of what's permitted/required
4. Disjunctions can introduce new preferences
   - So, !A $\nvDash$!A $\vee$!B
5. Imperatives are sensitive to information available
   - Preferences, by nature, are restricted to live options
6. Imperatives can scope under connectives
   - Dynamic semantics for connectives captures this

Thank you!

(Slides available at `http://williamstarr.net/research`)

References I

BROMBERGER, S (1966). 'Questions.' *The Journal of Philosophy*, **63(20)**: 597–606. URL `http://www.jstor.org/stable/2024253`.

GROENENDIJK, J (1999). 'The Logic of Interrogation: Classical Version.' In T MATTHEWS & D STROLOVITCH (eds.), *Proceedings from Semantics and Linguistic Theory IX*, 109–126. Ithaca, NY: Cornell University.

HAMBLIN, CL (1958). 'Questions.' *Australasian Journal of Philosophy*, **36**: 159–168.

HAN, CH (2001). 'Force, negation and imperatives.' *Linguistic Review*, **18(4)**: 289–325.

HANSSON, SO & GRÜNE-YANOFF, T (2009). 'Preferences.' In EN ZALTA (ed.), *The Stanford Encyclopedia of Philosophy*, spring 2009 edn. URL `http://plato.stanford.edu/archives/spr2009/entries/preferences/`.

HULSTIJN, J (1997). 'Structured Information States: Raising and Resolving Issues.' In A BENZ & G JÄGER (eds.), *Proceedings of MunDial97*, 99–117. Munich: Unversity of Munich. URL `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.26.6905`.

LEWIS, DK (1979). 'Scorekeeping in a Language Game.' *Journal of Philosophical Logic*, **8(3)**: 339–359.

References II

NEWELL, A (1992). *Unified Theories of Cognition.* Cambridge, Massachusetts: Harvard University Press.

RAMSEY, FP (1931). 'Truth and Probability.' In R BRAITHWAITE (ed.), *The Foundations of Mathematics: Collected Papers of Frank P. Ramsey*, 156–198. London: Routledge and Kegan Paul.

ROBERTS, C (1996). 'Information Structure in Discourse: Towards an Integrated Formal Theory of Pragmatics.' *Tech. rep.*, Linguistics Department, The Ohio State University, Columbus, Ohio. OSU Working Papers in Linguistics, volume 49, Jae-Hak Yoon and Andreas Kathol, editors., URL `http://www.ling.ohio-state.edu/~croberts/infostr.pdf`.

ROSS, A (1944). 'Imperatives and Logic.' *Philosophy of Science*, **11(1)**: 30–46. URL `http://www.jstor.org/stable/184888`.

STALNAKER, RC (1978). 'Assertion.' In P COLE (ed.), *Syntax and Semantics 9: Pragmatics*, 315–332. New York: Academic Press. References to reprint in Stalnaker (1999).

STALNAKER, RC (1999). *Context and Content: Essays on Intentionality in Speech and Thought.* Oxford: Oxford University Press.

STARR, W (to appear). 'Expressing *Must* and *May*.' *The Bulletin of Symbolic Logic.*

References III

VELTMAN, F (1996). 'Defaults in Update Semantics.' *Journal of Philosophical Logic*, **25(3)**: 221–261. URL `http://www.pgrim.org/philosophersannual/xix/velt/index.htm`.