A Preference Semantics for Imperatives

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Outline

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Observation 1
Ross, Disjunction, Consequence and Imperatives

- If what (1a) says is true, then what (1b) says is true

(1) a. Kathy posted the letter
b. Kathy posted the letter or Kathy burnt the letter

  - Classical semantics predicts this: $P \vDash P \vee B$

- Suppose that what (2a) commands is required
- Does it follow that what (2b) commands is required?

(2) a. Kathy, post the letter!
b. Kathy, post the letter or burn the letter!

- Ross' (1944) Puzzle: $P \vDash P \vee B$ but $!P \nvDash !P \vee !B$
- Proposition being true $\neq$ command being required?

Observation 1
Ross and Imperative Consequence

- Proposition being true $\neq$ command being required?
- Maybe:
  - Propositions are true, commands are satisfied
- Then imperative consequence is satisfaction-preservation
- So maybe $!P \vDash !P \vee !B$
- Maybe talk of requirement was pragmatic noise...

Against Satisfaction Consequence

1. Correct propositions are true
2. Correct commands are?
   - Satisfied $\times$; Required $\checkmark$

Observation 1

Imperative Consequence is not about Satisfaction

Fact 1: !P ⊭ May B

- President's command:
  (3) Will, post the letter!
- I cannot infer that
  (4) I may burn the letter

Against Satisfaction Consequence

- If imperative consequence is about satisfaction:
  - !P ⊨ !P ∨ !B
- Consequence is transitive:
  - !P ⊨ May B × ×

Fact 2: !P ∨ !B ⊨ May B

- The president's command:
  (5) Will, post the letter or burn the letter!
- I can infer:
  (6) I may burn the letter

Observation 1

Foreshadowing

Generalized Consequence

- An agent which accepts the premises has implicitly accepted the conclusion
  - Declaratives: After accepting premises, accepting conclusion provides no new information
  - Imperatives: After accepting premises, accepting conclusion provides no new permission
- Different kinds of sentence, different kinds of acceptance

Consequence in Dynamic Semantics

The generalized definition can be formulated with a dynamic semantics (Veltman 1996)

Observation 2

Felicity, Context & Information

(7) # Unicorns don't exist. Bring me a unicorn!

(8) # The door is open. Open the door!

Relatedly:

(9) a. I don't have a brother.
b. # If I had a brother, call him!

Generalization

The felicity of imperatives depends on the mutual information against which they are issued. Specifically, the possibility of the action they proffer must be open.

Bonus for: saying *why* imperatives are about open actions

Observation 3

Imperatives Scope Under Connectives

(10) Go home and I'll go to the grocery store.
a. **Assertion Conditional**:
Go home! And **if** you do, I'll go to the store
b. **Sequenced**:
I'll go to the grocery store and you go home
c. **Command Conditional**:
If you go home, I'll go to the grocery store
(And, you know what happens when I shop!)

- Sequenced requires imperative to scope under *and*
- Arguably, same point holds for conditional imperative:
  (11) If you're sleepy, drink coffee!

Preference, Rationality & Context

Information

- Informational contents (*propositions*) are sets of possible worlds
  - These sets distinguish ways world might be (worlds in the set) from ways it isn't (worlds excluded from set)
- One informational content is particularly useful for understanding how linguistic interactions unfold:

Contextual Possibilities (c)

As communication and inquiry unfold, a body of information accumulates. Think of this information as what the agents are mutually taking for granted in some way. I call the set of worlds embodying this information c, short for *contextual possibilities*. (Stalnaker 1978; Lewis 1979)

Preference, Rationality & Context

Information and the Process of Inquiry

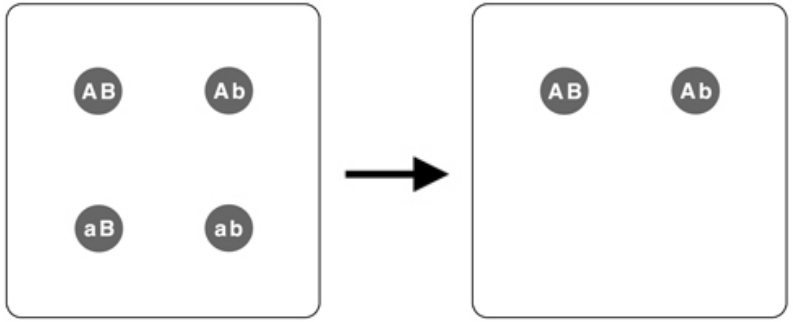


Figure: Accepting the information that A

- Inquiry progresses by gaining information, i.e. the elimination of worlds.
- $\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\} \Rightarrow \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$

Preference, Rationality & Context

Issues

- It's not just information that accumulates in communication and inquiry (Bromberger 1966)
- There are issues (e.g. Hamblin 1958; Roberts 1996).
- They can be thought of as ways of grouping worlds in c into competing alternative propositions.

Alternatives (C) (e.g. Groenendijk 1999)

Alternatives represent open, competing propositions the agents are concerned with deciding between; their **issues**. Formally, this grouping of c may be identified with a set of sets of worlds; call it C. There is no need to also keep track of c: it is just the union of all the alternatives in C.

Preference, Rationality & Context

Issues and Inquiry

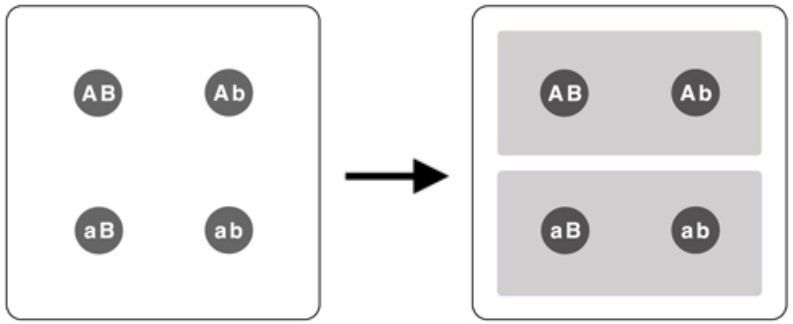


Figure: Recognizing the issue whether A

- Inquiry also progresses by recognizing issues, i.e. introducing alternatives
- $\{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\} \Rightarrow \{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\}$

Preference, Rationality & Context

Preferences

- Agents not only gather information and identify competing alternatives, they form **preferences** regarding those alternatives
- Central to **decision theoretic** approaches to rational choice, as applied in philosophy, AI and economics (e.g. Ramsey 1931; Newell 1992)
- Of relevance here: the preferences being mutually taken for granted for the purposes of an interaction
  - Parallel to Stalnaker's common ground

Preference, Rationality & Context

Preferences

- A body of preferences can be represented as a binary **preference relation** on the alternatives
- I.e. a set of pairs of propositions constructed from c

Preference State (R)

- R: binary relation on alternatives (open propositions)
- $R(a, a')$: a is preferred to a'
- Each pair in R is called a *preference*
- Set of (non-empty) alternatives over which R is defined: issues at stake in R, C_R
- Set of worlds among those alternatives: the contextual possibilities written c_R

Preference, Rationality & Context

Information in a Preference State

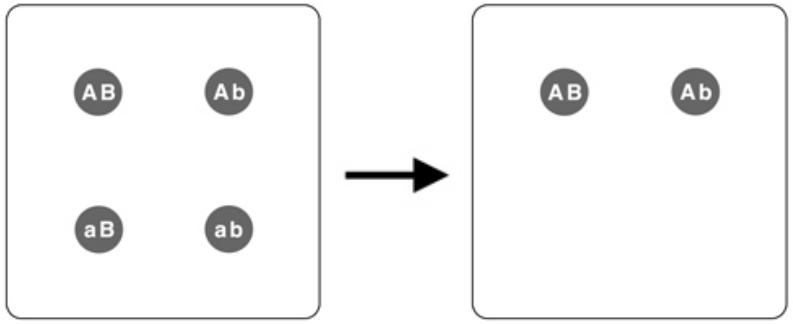


Figure: Accepting the information that A

- $\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{b}}\} \Rightarrow \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$
- $\{\, \langle \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{b}}\}, \varnothing \rangle \,\} \Rightarrow \{\, \langle \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \varnothing \rangle \,\}$

Preference, Rationality & Context

Issues in a Preference State

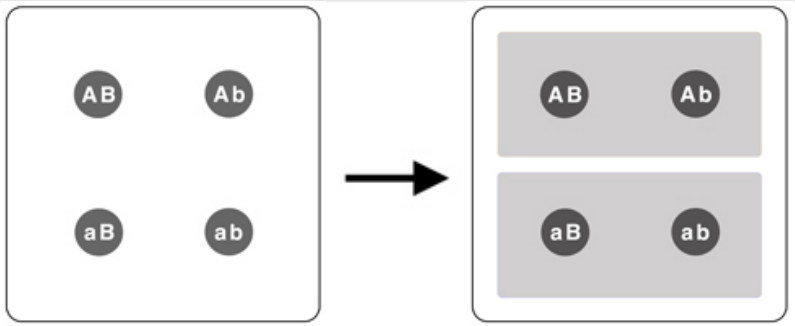


Figure: Recognizing the issue whether A

- $\{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\} \Rightarrow \{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\}$
- $\{\, \langle \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}, \varnothing \rangle \,\}$
  $\Rightarrow \{\, \langle \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \varnothing \rangle, \langle \{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}, \varnothing \rangle \,\}$

Preference, Rationality & Context

Preference and Inquiry

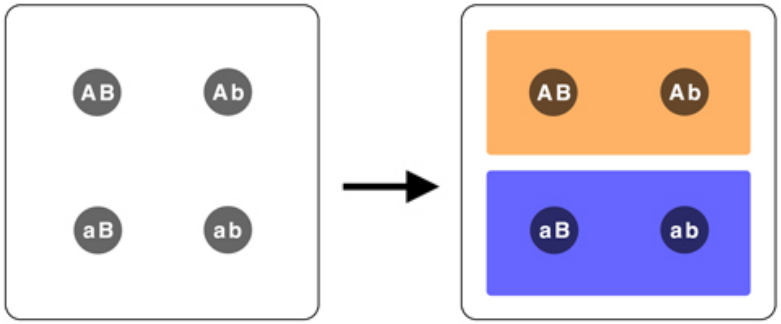


Figure: Coming to prefer A (to ¬A)

- $\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle\}$
  $\Rightarrow \{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle\}$

Preference, Rationality & Context

Preference and Inquiry

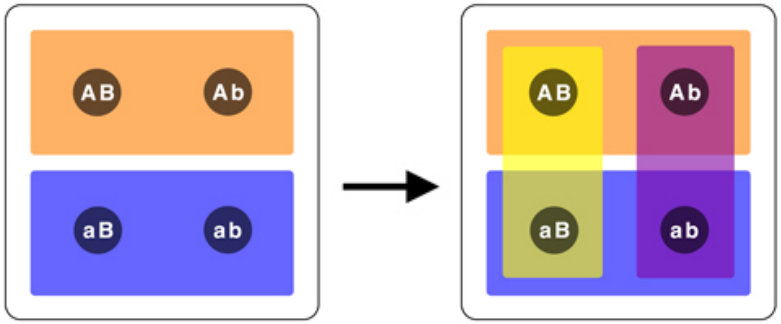


Figure: Adding (separate) preference for B to preference for A

- $\{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle\} \Rightarrow$
  $\{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle, \langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$

Preference, Rationality & Context

Using Preference to Make Rational Choices

- Given preference relation, which alternatives are best?
- How do you use preferences to decide what to do?
- In decision theory, this takes the form of defining a choice function (Hansson & Grüne-Yanoff 2009)
- A choice function Ch maps a preference state R to the set of best alternatives according to R

Proposal: Choice, Permission, Requirement

1. $Ch(R)$ are the alternatives permissible according to R
2. Required by R: unique alternative permitted by R

Preference, Rationality & Context

The Choice Function: Logical Weak Dominance

Which Alternatives are Best?

1. Competition between **preferred alternatives** $P(R)$
   - Left member in some pair
2. If preferred alternative a is entailed another preferred one, then a is out
3. If a entails a dispreferred alternative, a is out

Choice: Formally

$$Ch(R) = \{a \in P(R) \mid \nexists a' \in P(R) : a' \subset a \;\&\; \nexists a' \in D(R) : a \subseteq a'\}$$

[$D(R)$: dispreferred alternatives]

Preference, Rationality & Context

How Choice Works: An Example

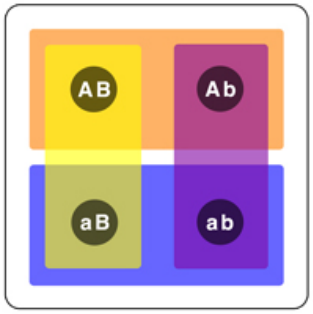
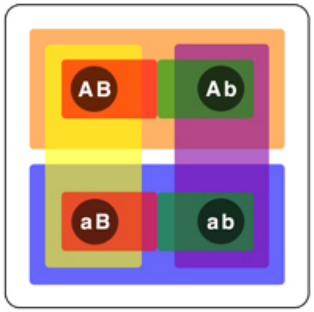


Figure: Preference for A with (separate) preference for B

- $\{\langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle, \langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$
- Two **preferred** (warm) alternatives, orange and yellow
- Neither entails the other nor dispreferred (cold) alt.
- So $Ch(R) = \{\{w_{AB}, w_{Ab}\}, \{w_{AB}, w_{aB}\}\}$

Preference, Rationality & Context

How Choice Works: A More Complex Example



- 4 pref. alt's: yellow, orange, reds
- Yellow is out: reds entail it
- Orange is out: top red entails it
- Bottom red is out: it entails blue, which is a dispreferred alt
- Unique best alternative: top red
- $A \wedge B$ is required

Figure: Pref A and B

$$\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle, \langle\{w_{AB}\}, \{w_{Ab}\}\rangle, \langle\{w_{aB}\}, \{w_{ab}\}\rangle, \langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$$

Preference, Rationality & Context

What Must Preferences Be Like for Choice to Guarantee Results?

Exclusivity

- $\forall a, a' : a \cap a' = \varnothing$ if $R(a, a')$
- *When you strictly prefer one thing to another, the two can't be compatible.*

No Absurdity

- $\forall a \neq \varnothing : \langle a, \varnothing\rangle \in R \;\&\; \langle \varnothing, a\rangle \notin R$
- *Always prefer non-absurd alternatives to absurd one.*

Irreflexivity

- $\forall a : \langle a, a'\rangle \notin R$ if $a' \subseteq a$
- *You can't strictly prefer an alternative to something that entails it.*

Preference, Rationality & Context

These Constraints on Preferences are Pragmatic

Semantics, Pragmatics & Irrational Preferences

- Words can get us into irrational preference states
  - So none of these axioms are enforced in the semantics
- Rather, recognizing their satisfaction and frustration is part of pragmatics
- Grice: pragmatics is about general rational cooperation
- Decision Theory: rational agents follow preference axioms

The Semantics: some preliminaries

Radicals & Worlds

Radicals (Informational Core)

- Basic sentences: mood marker + radical, e.g. $!\rho$
  - Mood markers: $!, \triangleright, ?$
  - Atomic radicals: $\mathsf{A}, \mathsf{B}, \mathsf{C}$, etc.
  - Logically complex radicals: $\neg\rho, \rho_1 \wedge \rho_2, \rho_1 \vee \rho_2$

Worlds

A possible world is a function which maps atomic radicals to a unique truth-value, 1 or 0

- Dynamic Meaning: function from contents to contents
- $R[\phi] = R'$: R' is the result of applying ϕ to R (Veltman 1996)

The Semantics

Atomic Radical Semantics

Radical Semantics

- $c[\mathsf{A}] = \{w \in c \mid w(\mathsf{A}) = 1\}$, for any atomic radical A

- Subsentential semantics
- Filters alternatives for worlds where radical is true

Connective Semantics (Heim, Veltman)

- $c[\neg\rho] = c - c[\rho]$
- $c[\rho_1 \wedge \rho_2] = (c[\rho_1])[\rho_2]$
- $c[\rho_1 \vee \rho_2] = c[\rho_1] \cup c[\rho_2]$
- $R[\phi \wedge \psi] = (R[\phi])[\psi]$
- $R[\phi \vee \psi] = R[\phi] \cup R[\psi]$

The Semantics

Imperative Semantics

Imperative Semantics

$R[!\rho] = R \cup \{\langle a[\rho], a - a[\rho]\rangle \mid a \in A_R\}$

- A_R: non-empty R-alternatives, plus their union c_R

This amounts to a three-step process:

1. Admit all of the preferences in R
2. **Local Preferences**: Take each incoming non-empty alternative a and introduce a preference for the ρ-worlds in a over the non-ρ-worlds in a
3. **Global Preference**: Introduce a preference for all of the ρ-worlds in c_R over the non-ρ-worlds

The Semantics

A Simple Example

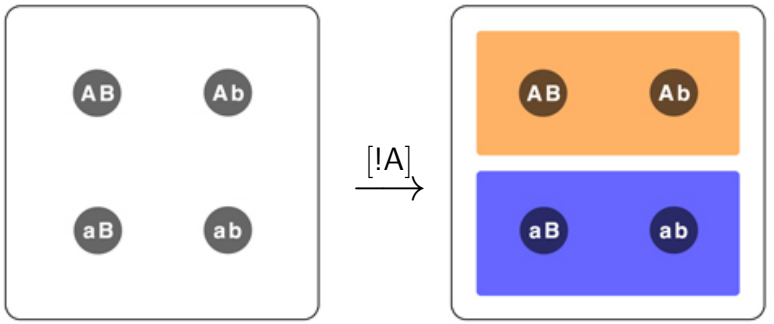


Figure: R to $R[!\mathsf{A}]$

$R = \{\langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}, \varnothing\rangle\} \Rightarrow$
$\{\ \langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}, \varnothing\rangle, \langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\rangle\ \}$

The Semantics
A Complex Example

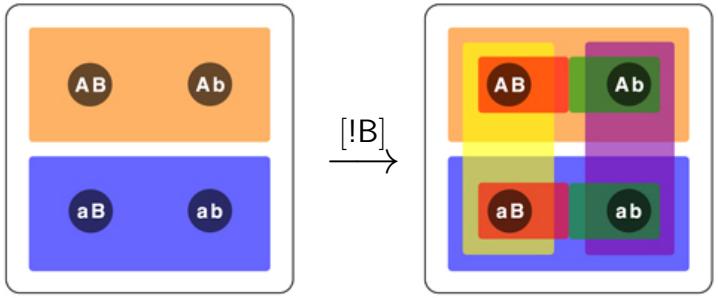


Figure: $R[!A]$ to $R[!A][!B]$

$$\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle\} \Rightarrow$$
$$\{\langle\{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing\rangle, \langle\{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\}\rangle,$$
$$\langle\{w_{aB}\}, \{w_{ab}\}\rangle, \langle\{w_{AB}\}, \{w_{Ab}\}\rangle,$$
$$\langle\{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\}\rangle\}$$

The Semantics
A Complex Example

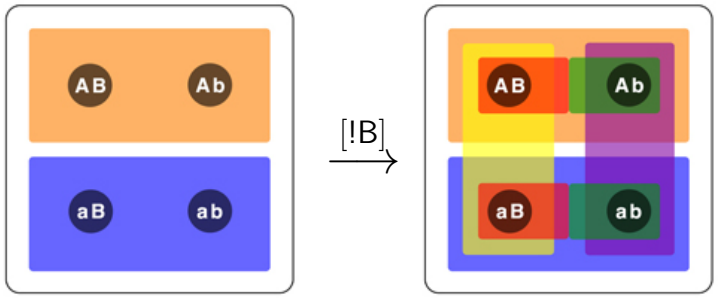


Figure: $R[!A]$ to $(R[!A])[!B]$

- Recall $R[!A \wedge !B] = (R[!A])[!B]$
- So this is the interpretation of conjoined imperatives
- Let's mix in a declarative conjunct...

The Semantics: Observation 3
Mixing Moods

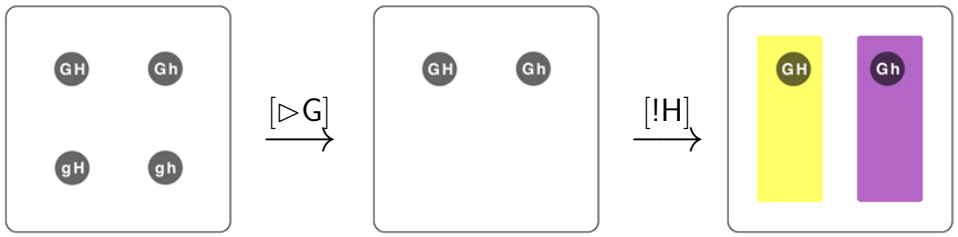


Figure: $(R[\triangleright G])[!H]$

- *I'll go to the grocery store and you go home*: $\triangleright G \wedge !H$
- $R[\triangleright G \wedge !H] = (R[\triangleright G])[!H]$

The Semantics: Observation 2
Felicity, Context and Information

Figure: $(R[\triangleright \neg U])[!B]$

- *Unicorns don't exist. Bring me a Unicorn*:
  - $(R[\triangleright \neg U])[!B]$
- This is a very irrational preference: the absurd!
  - Hence (pragmatically) infelicitous

The Semantics: Observation 1

We want !A ⊭ !A ∨ !B

Informational Consequence (Veltman 1996)

$\phi_1, \ldots, \phi_n \vDash \psi$ iff $\forall c : c[\phi_1] \cdots [\phi_n] = (c[\phi_1] \cdots [\phi_n])[\psi]$

- After accepting premises, accepting conclusion provides no new information.

Proposal: Choice Consequence

$\phi_1, \ldots, \phi_n \vDash \psi$ iff

$$\forall R : Ch(R[\phi_1] \cdots [\phi_n]) = Ch(\,(R[\phi_1] \cdots [\phi_n])[\psi]\,)$$

- After accepting premises, accepting conclusion provides no new permissions/requirements

The Semantics: Observation 1

We want !A ⊭ !A ∨ !B

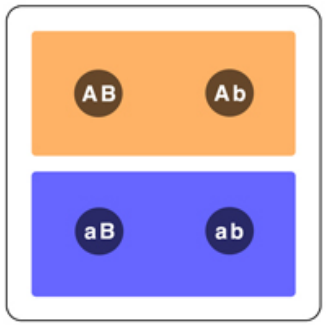


Figure: R(!A)

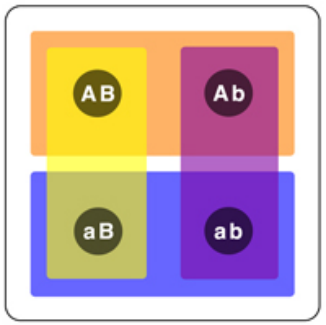


Figure: R(!A ∨ !B)

- Why: !A ⊭ !A ∨ !B
  - Disjunctive imperatives create more permissions
- But consequence is about preserving permissions

Conclusion

Three Observations and a Semantics

Summary

1. Imperatives introduce preferences
2. Preferences are used to determine what's permitted/required
3. Imperative consequence: preservation of what's permitted/required
4. Disjunctions can introduce new preferences
   - So, !A ⊭ !A ∨ !B
5. Imperatives are sensitive to information available
   - Preferences, by nature, are restricted to live options
6. Imperatives can scope under connectives
   - Dynamic semantics for connectives captures this

Thank you!

(Slides available at `http://williamstarr.net/research`)

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