The Logic of Atomic Sentences

Proof & Logical Consequence

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Announcements

For 01.27

1. HW1 is due now
2. HW2 is due next Tuesday

Outline

Previously On. . .

Phil 201

- We learned how to represent some simple English sentences in FOL
  - Example: *Mars is red* $\leadsto$ Red(mars)
- But remember **why** we did this:
  - Arguments are phrased in language that often obscures their important logical properties
  - So, we are learning how to represent them in a more useful way: FOL
  - Since arguments contain premises & conclusions we needed to learn how to represent those premises & conclusions in FOL
- We also learned how to carve up and represent arguments using the Fitch Format

Today
An Overview

- Today, we will:
  1. Learn what it takes for an argument to be good
     - That is, what it takes for an inference to be correct
  2. Learn how to show that an argument is good
     - This will involve learning about the idea of a proof
- However, throughout we will focus on arguments containing **atomic sentences**
- Later in the course we will extend our theories of inference and proof to a larger class of arguments

Logical Consequence & Validity
The Definitions

- The first property good arguments have is what we'll call being logically valid

Logical Validity & Consequence

1. An argument is logically valid if and only if there is no way of making the premises true that does not make the conclusion true as well
2. In general, we say that one claim is a logical consequence of another if and only if there is no way the latter could be true without the former also being true

- In a valid argument the truth of the premises guarantees the truth of the conclusion

Logical Consequence & Validity
Some Examples

Example 1

1	Jay and Kay live on the same street
2	Kay and Elle live on the same street
3	Jay and Elle live on the same street

- Is this a logically valid argument?
  - Yes:
    - Assuming 1 & 2, there's no street that Jay can live on which is not Elle's street
    - That is, there's no way for 1 & 2 to be **true** without 3 being true

Logical Consequence & Validity
Some Examples

Example 2

1	All actors who win Academy Awards are famous
2	Harrison Ford has never won an Academy Award
3	Harrison Ford is not famous

- Is this a logically valid argument? No:
  - 1 requires only that every actor who wins an Academy award be famous
    - But, it's consistent with this for there to be famous people who don't win an Academy Award
    - So it's consistent with 1 to assume that Ford hasn't won an Academy Award **and** that Ford is famous
  - So it's possible for 1 & 2 to be true w/o 3 being true

Logical Consequence & Validity
Being Compelled

- So, in a logically valid argument there's no way for the premises to be true without the conclusion being true
- But what **exactly** does being logically valid have to do with an argument's being compelling?

Beyond Consequence & Validity
That's Not the Whole Story

- Being logically valid is a big part of what it takes for an argument to be compelling, but it isn't the whole story

A Valid Argument That Isn't Compelling

1	All grandmothers are omnipotent
2	Letticia is a grandmother
3	Letticia is omnipotent

- If I offered you this argument would you be compelled to believe that my grandmother Letticia is omnipotent?
  - Of course not! But why?

Beyond Consequence & Validity
What's Missing

A Valid Argument That Isn't Compelling

1	All grandmothers are omnipotent
2	Letticia is a grandmother
3	Letticia is omnipotent

- The argument **is** valid , but remember what that shows:
  - If you accept the premises, you must accept the conclusion
  - But premise 1 is ridiculous, so you'd never accept it!

Beyond Consequence & Validity
It's *Soundness*

- So it looks like a good argument is not only one that is valid
  - It's premises must also be true
- This is a property called soundness
- Let's take a closer look

Soundness

The Definition

Soundness

An argument is sound if and only if it is logically valid **and** its premises are true

- Soundness requires two things
  1. Validity
  2. True premises

Soundness

Getting Back to Granny

The Granny Argument

1	All grandmothers are omnipotent
2	Letticia is a grandmother
3	Letticia is omnipotent

- Again, the argument is valid
- Is it sound?
  - No, premise 1 is false — unfortunately, grannies are not all-powerful

Soundness

Pushing Our Understanding

Example 2

1	All actors who win Academy Awards are famous
2	Harrison Ford has never won an Academy Award
3	Harrison Ford is not famous

- Is this argument sound?
  - It's premises are true! Does that mean it's sound?
  - No! Soundness requires validity as well, and recall from before that this argument isn't valid

To solidify our grasp of soundness & validity, let's work **exercise 2.4** (p.46)

In Class Exercise

Exercise 2.7

Break up into groups of 6 or fewer and do **Exercise 2.7** (p.53).

After 10 minutes, I'll call on someone to give their group's answers

Proof
Showing Validity

- Our description of logical consequence is great in theory
  - But, it doesn't give us any specific tools for actually showing that a given argument is valid
- In our simple examples it was fairly easy to tell whether or not the arguments were valid
  - But, for most interesting arguments this issue cannot be decided so easily
- Today, we'll begin to learn the more precise & powerful techniques for doing this that modern logic offers
- The key notion here will be that of proof

Proof
What is it?

Proof

A proof is a step-by-step demonstration which shows that a conclusion C must be true in any circumstance where some premises $P_1, \ldots, P_n$ are true

1. The step-by-step demonstration of C can proceed through intermediate conclusions
2. It may not be obvious how to show C from P_1 and P_2, but it may be obvious how to show C from some other claim Q that **is** an obvious consequence of P_1 and P_2
3. Each step must provide incontrovertible evidence for the next

Proofs
What they Accomplish

What's so Insightful about Proofs?

The insight behind proofs is that by breaking up an argument into a series of steps one can determine whether or not it is valid by determining whether or not each step is correct

- By breaking an argument down into a full proof, we reduce a very hard question:
  - *Is this argument valid?*

  to a much easier one:
  - *Is each step of this proof correct?*

Proof
An Example Argument

Argument 3

| Vin Diesel is a man
| All men are mortal
| Everyone who will die sometimes worries about it
|---
| Vin Diesel sometimes worries about dying

- It's not exactly obvious whether or not Argument 3 is valid, so let's try to construct a proof

Proof
An Example Proof

Proof that Argument 3 is Valid

Since Vin is a man & all men are mortal, it follows that Vin is mortal. But all mortals will eventually die, since that is what it means to be mortal. So Vin will eventually die. But we are given that everyone who will eventually die sometimes worries about it. Hence Vin sometimes worries about dying.

- This is a step-by-step demonstration that the conclusion of Argument 3 must be true if the 3 premises of Argument 3 are true
- Each step consists of a simple, obvious, valid inference

Proof
Steps

- By chaining together obvious steps we get a proof
  - But what exactly were these steps?
  - Why were they so obvious?
  - Where do they come from?
- Let's try to answer these questions

Proof
A Simpler Example

Argument 4

Superman is Clark Kent

Superman is from Krypton

Clark Kent is from Krypton

Proof

Since superman **is** Clark Kent, whatever holds of Superman also holds of Clark Kent. We are given that Superman is from Krypton, so it must be the case that Clark Kent is from Krypton.

Proofs
Steps

- In our proof, how did we justify the move from *Superman is Clark Kent & Superman is from Krypton* to *Kent Clark is from Krypton*?
- We said: *Since Superman is Clark Kent, whatever holds of Superman also holds of Clark Kent*
- This is an instance of a more general principle called the indiscernibility of identicals

Indiscernibility of Identicals

If a is b, then whatever is true of a is also true of b (where 'a' and 'b' are names)

Proof
The Indiscernibility of Identicals

Indiscernibility of Identicals

If a is b, then whatever is true of a is also true of b (where 'a' and 'b' are names)

- This is a generalization about what is means for *a is b* to be true

Proof
The Moral of this Tangent

The Nature of Steps

Each step of a proof will appeal to certain facts about the meaning of the vocabulary involved. These facts are what we implicitly appeal to when we say 'that step is obviously right'

- In the case of our proof of Argument 4, it was a fact about the meaning of *is*:
  - Namely the Indiscernibility of Identicals
- Similar facts underlie the steps in our proof that Argument 3 is valid
- To solidify this fact, let's look one more argument

Proof
One More Example

Argument 5

b is to the right of c
d is to the left of e
b is d
---
c is left of e

Proof

We are told that b is to the right of c. So c must be to the left of b, since *right of* & *left of* are **inverses** of each other. And since $b = d$, c is left of d by the Indiscernibility of Identicals. But we are also told that d is left of e, and consequently c is to the left of e, by the textbftransitivity of *left of*. Done.

Proof
How the Proof Worked

In two steps of our proof, we appealed to facts about the meaning of *left of* & *right of*:

1. *left of* & *right of* are inverse relations
   - By *inverse* I mean the relations are opposites, so if you invert the order of the names they say the same thing:
     - *a is right of b* means the same as *b is left of a* and vice versa
2. *left of* is transitive:
   - If a is left of b & b is left of c, then a is left of c as well

Proof
A Little Bit More About Steps

- In addition to properties like inversion and transitivity there are other important properties that some predicates exhibit:
  1. Symmetry (p.52 of *LPL*)
  2. Reflexivity (p.52 of *LPL*)
- You should know what these properties are!

Proofs
Summary

1. What it takes for an argument to be good (correct):
   - Soundness (= Validity + True Premises)
2. How to demonstrate that an inference is valid: a proof
3. A proof breaks a non-obvious inference down into a series of trivial, obvious steps which lead you from the premises to the conclusion
   - These *steps* are based on facts about the *meaning* of the terms involved

Proofs
Where We Are

- We have a basic grasp of how to write out simple proofs in English
- But, there's two things we haven't done:
  1. Written many proofs that use steps other than the Indiscernibility of Identicals
  2. Figured out all the rules for predicates involved in proofs we might want to write
- We're not going to go do 2, it would take forever (literally) & would be boring
- Instead, we've looked at:
  - Some important steps involving *is*
  - Some abstract properties to look at when thinking about the meaning of predicates
    - Inverses, Transitivity, Reflexivity, Symmetry

Proofs
Where We Are Going

- So far, we've been writing proofs out in ordinary English
- But, there's another way of doing it that's worth knowing
- This other way involves developing what's called a formal system of deduction
- Proofs in a formal system of deduction (aka *formal proofs*) aren't any more rigorous
- They're different stylistically and useful for various purposes

Formal Proofs

What They Are Good For

- Formal proofs are useful for a number of reasons:
  1. They format proofs in a way that makes their structure more transparent
  2. Every single step of the proof is included and each fact that is used to justify each step is explicitly cited
  3. When formulated formally, a proof can be checked or performed by a computer
  4. Mathematicians & logicians can prove facts about what is provable by proving facts about a formal system of deduction

Homework 1

Due Tuesday 02.02

HW1

- 2.1, 2.2, 2.6, 2.8, 2.20
- Due by Tuesday 02.02